Modal Identification and Finite Element Model Updating of Metsovo Bridge

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Outline

I. BAYESIAN FRAMEWORK FOR FINITE ELEMENT MODEL UPDATING

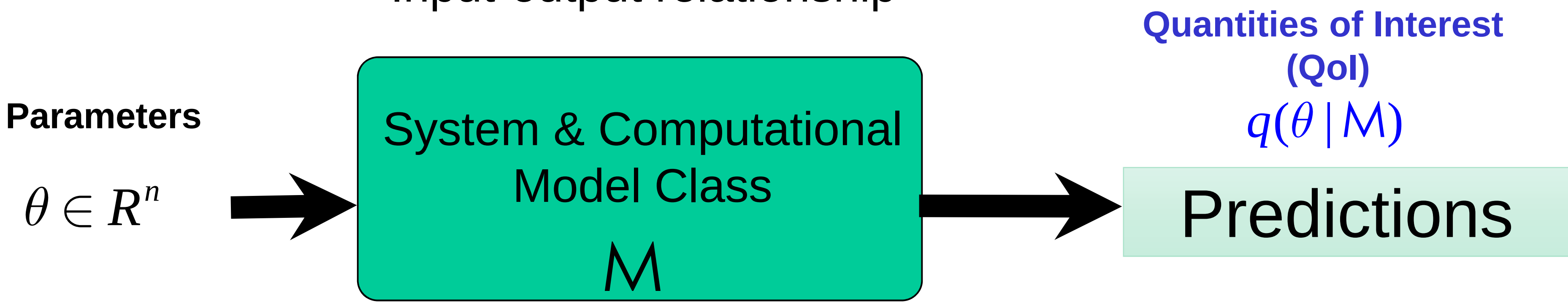
II. BAYESIAN TOOLS: COMPUTATIONAL CHALLENGES

III. APPLICATION: METSOVO BRIDGE

- Modal Identification
- Finite Element Model Updating

Structural Dynamics Model Classes

Input-output relationship



- **Nonlinear model classes – Governing Equation of Motion (e.g. FE model)**

$$M(\theta)\ddot{x}(t) + g(x(t), \dot{x}(t); \theta) = L(\theta)\,p(t)$$

$$q(\theta \mid \mathrm{M}) = Q(x, \dot{x}, \ddot{x}, \theta)$$

- **Linear model classes – Governing Equation of Motion**

$$M(\theta)\ddot{x}(t) + C(\theta)\dot{x}(t) + K(\theta)x(t) = L(\theta)\,p(t)$$

- **Linear model classes – Eigenproblem**

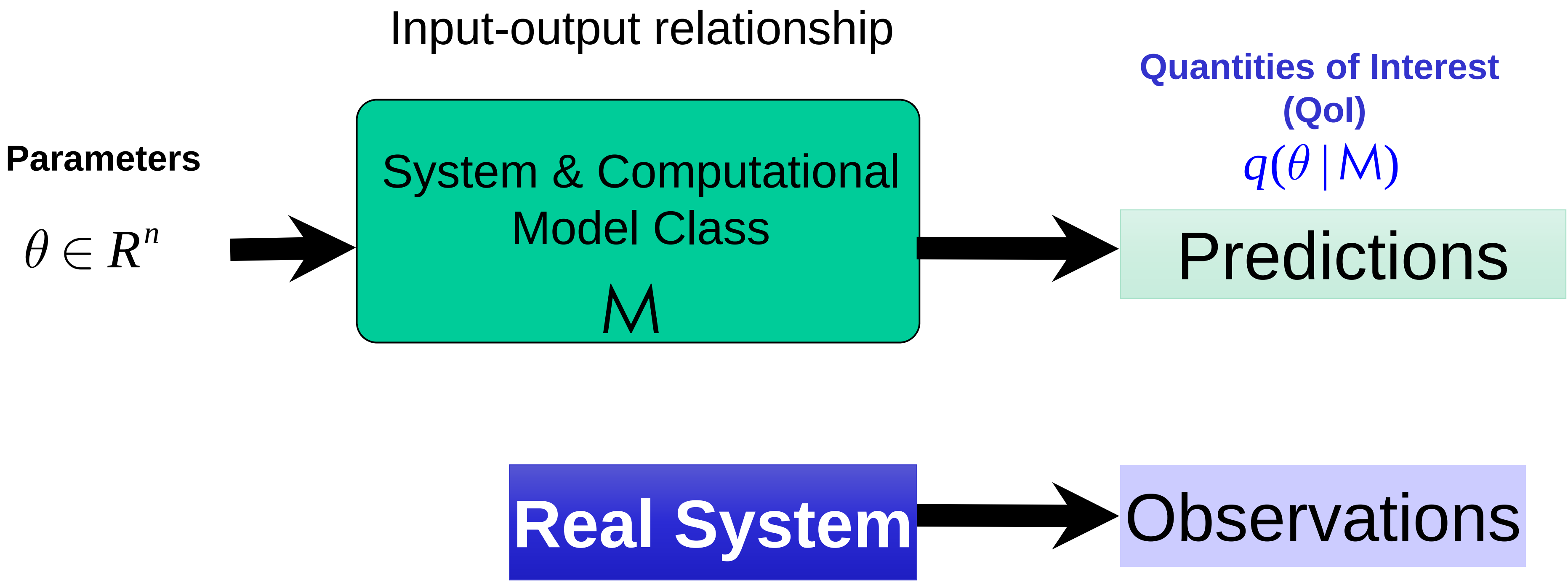
$$[K(\theta) - \omega^2 M(\theta)]\phi = 0$$

I.

Bayesian Framework for Finite Element Model Updating

(Bayesian Uncertainty Quantification)

Bayesian UQ+P



- Response time histories
- Modal frequencies & mode shapes
- Response spectra (nonlinear)

Used at component or system level to select and update uncertainty models

Bayesian UQ

Parameter Estimation

Posterior PDF — **Likelihood** — **Prior**

$$f(\theta \mid D, M) = \frac{f(D \mid \theta, M)\ \pi(\theta \mid M)}{f(D \mid M)}$$

Evidence

$$f(D \mid \theta, M) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2} [\hat{y} - q(\theta \mid M)]^T \Sigma^{-1}(\theta) [\hat{y} - q(\theta \mid M)] \right]$$

Bayesian UQ

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Model Class Selection

$$\Pr(M_i \mid D) = \frac{f(D \mid M_i)\ \Pr(M_i)}{f(D)}$$

Evidence

$$f(D \mid M_i) = \int f(D \mid \theta_i, M_i)\ \pi(\theta_i \mid M_i)\ d\theta_i$$

Bayesian UQ+P

Parameter Estimation

Likelihood — Prior

Posterior PDF

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Evidence

$$f(D \mid M_i) = \int f(D \mid \theta_i, M_i)\ \pi(\theta_i \mid M_i)\ d\theta_i$$

Posterior Robust Predictions

$$E[G_q(\theta) \mid D, M] = \int G_q(\theta \mid M)\ f(\theta \mid D, M)\ d\theta$$

Posterior PDF

- Mean
- Standard deviation
- Credible intervals
- Failure probability

Bayesian UQ

PROBABILISTIC MODEL of discrepancy between model predictions and data

Prediction Error:

$$\hat{y} = y(\theta \mid \mathrm{M}) + e$$

$$e = e^e + e^m$$

Principal of maximum entropy to select Normal distribution

Experimental Error: $e^e \sim N(0, \Sigma^e)$

Model Error: $e^m \sim N(0, \Sigma^m)$ → **Include in parameter set** θ

LIKELIHOOD: independent prediction errors $\Sigma(\theta) = \Sigma^e + \Sigma^m$

$$f(D \mid \theta, \mathrm{M}) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[\hat{y} - q(\theta \mid \mathrm{M})]^T \Sigma^{-1}(\theta)[\hat{y} - q(\theta \mid \mathrm{M})]\right]$$

Case 1: Modal Characteristics (Linear Models)

MEASUREMENTS: Modal frequencies and Modeshapes

$$\hat{\omega}_r^2 = \omega_r^2(\theta \,|\, \mathcal{M}) + \hat{\omega}_r^2 e_{\omega_r}$$

$$\underline{\hat{\varphi}}_r = \underline{\varphi}_r(\theta \,|\, \mathcal{M}) + \| \underline{\hat{\varphi}}_r \| \, \underline{e}_{\varphi_r}$$

PROBABILITY STRUCTURE OF PREDICTION ERROR

Modal Frequency Error: $e_{\omega_r} \sim N(0, \sigma_\omega^2)$

Modeshape Error: $\underline{e}_{\varphi_r} \sim N(\underline{0}, \sigma_\varphi^2 I)$

Include in parameter set θ

LIKELIHOOD: independent prediction errors

$$f(D \,|\, \theta, \mathcal{M}) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^{n/2}} \exp\left[-\frac{1}{2} J(\theta; \mathcal{M}) \right]$$

Case 1: Modal Characteristics (Linear Models)

LIKELIHOOD: independent prediction errors

$$f(D \mid \theta, \mathrm{M}) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^{n/2}} \exp\left[-\frac{1}{2} J(\theta; \mathrm{M})\right]$$

$$J(\theta; \mathrm{M}) = \frac{1}{\sigma_\omega^2} J_\omega(\theta) + \frac{1}{\sigma_\varphi^2} J_\varphi(\theta)$$

$$J_\omega(\theta) = \sum_{r=1}^{m} \left[\frac{\omega_r^2(\theta) - \hat{\omega}_r^2}{\hat{\omega}_r^2}\right]^2$$

$$J_\varphi(\theta) = \sum_{r=1}^{m} \frac{\| \beta_r \varphi_r(\theta) - \hat{\varphi}_r \|^2}{\| \hat{\varphi}_r \|^2} = \sum_{r=1}^{m} \left[1 - MAC^2(\varphi_r, \hat{\varphi}_r)\right]$$

$$\beta_r = \frac{\varphi_r^T(\theta)\hat{\varphi}_r}{\| \hat{\varphi}_r \|^2}$$

$$|\Sigma(\theta)| = \sigma_\omega^m \sigma_\varphi^{mN_0}$$

Case 2: Time Histories (Nonlinear Models)

MEASUREMENTS: Acceleration, displacement, strain time histories

$$\hat{y}_j(k) = y_j(k;\theta \,|\, \mathrm{M}) + \underline{e}_j(k)$$

Prediction Error:

$$e_j(k) = e_j^d(k) + e_j^m(k)$$

PROBABILITY STRUCTURE OF PREDICTION ERROR

Experimental Error: $e_j^d(k) \sim N(0, s_j^2 \,\|\, \hat{y}_j \,\|^2)$

Include in parameter set θ

Model/Parametric Error: $e_j^m(k) \sim N(0, \sigma_j^2 \,\|\, \hat{y}_j \,\|^2)$

LIKELIHOOD: independent prediction errors

$$f(D \,|\, \theta, \mathrm{M}) = \frac{\| \hat{y}_j \|^{-N_0 N/2}}{(2\pi)^{N_0 N/2} (s_j^2 + \sigma_j^2)^{N_0 N/2}} \exp\left[-\frac{1}{2} \sum_{j=1}^{N_0} \frac{\| \hat{y}_j \|^{-2}}{s_j^2 + \sigma_j^2} \sum_{k=1}^{N} [\hat{y}_j(k) - y_j(k;\theta \,|\, \mathrm{M})]^2 \right]$$

Case 3: Response Spectra (Nonlinear Models)

MEASUREMENTS: Acceleration & / or velocity & / or displacement Response Spectra

Prediction Error:

$$\hat{y} = y(\theta \,|\, \mathrm{M}) + e$$

$$e = e^d + e^c + e^m$$

PROBABILITY STRUCTURE OF PREDICTION ERROR

Experimental Error: $e^d \sim N(0, diag(s_r^2 \hat{y}_r^2))$

Computational Error: $e^c \sim N(0, diag(\lambda_r^2 \hat{y}_r^2))$

Model / Parametric Error: $e^m \sim N(0, diag(\sigma_r^2 \hat{y}_r^2))$

Include in parameter set θ

LIKELIHOOD: independent prediction errors $\Sigma(\theta) = diag[(s_r^2 + \lambda_r^2 + \sigma_r^2)\hat{y}_r^2)]$

$$f(D \,|\, \theta, \mathrm{M}) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^{n/2}} \exp\left[-\frac{1}{2}[\hat{y} - y(\theta \,|\, \mathrm{M})]^T \Sigma^{-1}(\theta)[\hat{y} - y(\theta \,|\, \mathrm{M})]\right]$$

II.
Bayesian Tools: Computational Challenges

AA: **ASYMPTOTIC APPROXIMATIONS**

SSA: **STOCHASTIC SIMULATION ALGORITHMS**

AA: Asymptotic Approximations

Parameter Estimation

Gaussian Posterior PDF

$$f(\theta \mid D, M) \approx N(\theta; \hat{\theta}, H^{-1}(\hat{\theta}))$$

Most Probable Model:

$$\hat{\theta} = \arg \min_{\theta} [-\ln f(\theta \mid D, M)]$$

Covariance:

$H(\theta)$ **is Hessian of** $-\ln f(\theta \mid D, M)$

AA: Asymptotic Approximations

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Model Class Selection

$$f(D \mid M) \approx c_0 \left(2\pi\right)^{N_\theta/2} \frac{\pi_\theta(\hat{\theta} \mid M)\, f(D \mid \hat{\theta}, M)}{\sqrt{\det H(\hat{\theta})}}$$

Evidence

AA: Asymptotic Approximations

Parameter Estimation

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Evidence

Posterior Robust Predictions

$$E[G_q(\theta) \mid D, M] = G_q(\hat{\theta}_G \mid M) \frac{f(D \mid \hat{\theta}_G, M)\, \pi(\hat{\theta}_G \mid M)}{f(D \mid \hat{\theta}, M)\, \pi(\hat{\theta} \mid M)} \frac{\sqrt{\det H(\hat{\theta})}}{\sqrt{\det H_G(\hat{\theta}_G)}} \text{ x}$$

$$\{1 + O(N^{-2})\}$$

SA: Sampling Algorithms

Parameter Estimation

Samples drawn from Posterior PDF (e.g. using variants of MCMC, Transitional MCMC)

$$\theta^{(i)} \sim f(\theta \mid D, M)$$

Model Class Selection

- **Estimation of EVIDENCE requires special algorithms and extra system simulations**
- **Transitional MCMC (TMCMC): EVIDENCE does not require extra system simulations**

Posterior Robust Predictions

A. Sample Estimate for mean, variance, etc.

$$E[G_q(\theta) \mid D, M] \approx \frac{1}{N}\sum_{i=1}^{N} G_q(\theta^{(i)} \mid M)$$

Conditional

B. Failure Probability Estimate

MCMC samples are used at first stage of Subset simulation
[Jensen, Vergara, Papadimitriou, Milas, CMAME, 2013]

COMPUTATIONAL CHALLENGES for complex models

- **Model Complexity:**

  - Very large number of model DOFs

  - Model nonlinearities (often localized)

- **Excessive computational effort due to a moderate to very large number of structural/system model re-analyses required**

AA vs SA for Serial and Parallel Implementation

	AA-Gradient-based Optimization	AA-Stochastic Optimization (CMA-ES)	SA-Sampling: TMCMC multi-chain
- # of Model Runs	Moderate	Very large	Very large
- Adjoint Techniques	Yes	No	No, but useful
- Model Intrusiveness	Yes	No	No
- System Models	Limited Classes	All classes	All classes
- Experimental Data	Limited types	All classes	All classes

Time to Solution (Parameter Estimation + Evidence)

- Serial	Iterations x 2	Generations x Population	MC samples
- Parallel	Iterations x 2	Generations	Stages x Samples in Longest Chain

Time to Solution (Uncertainty Propagation)

- Serial	Iterations x2x N_{QoI}	Generations x N_{QoI}	
- Parallel	Iterations x 2	Generations	None

AA & SA

Techniques for reducing computational effort

- **LEVEL OF SA ALGORITHM: Multi-Chain TMCMC**

  HPC: Parallel Implementation to efficiently distribute repeated system simulations to GPUs & multi-core CPUs **[Hatjidoukas et al. J Computational Physics 2015]**

AA & SA

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  Surrogate Models within TMCMC to drastically reduce the number of full model runs

  Multi-Chain/Parallel/Surrogate TMCMC: X-TMCMC

  [Angelikopoulos, Papadimitriou & Koumoutsakos, J Chemical Physics 2012]

  [Angelikopoulos, Papadimitriou & Koumoutsakos, CMAME 2015]

AA & SA

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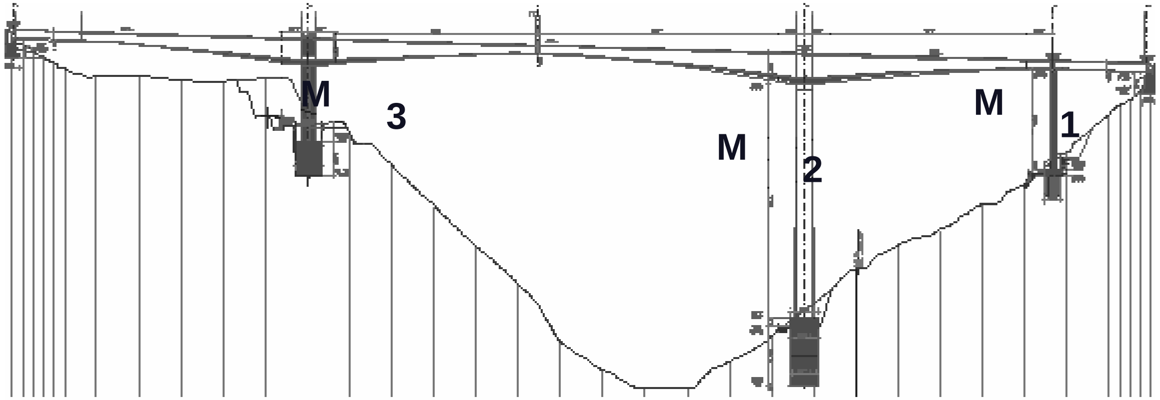
- **MODEL LEVEL (AA & SA): Model reduction**

  Component mode synthesis (consistent with parameterization scheme) to drastically reduce the size of system and computational effort without sacrificing in accuracy **[Papadimitriou & Papadioti, Computer & Structures 2013; Jensen et al., CMAME, 2014]**

3. Model Reduction

Model Updating using CMS

R/C Bridge at Metsovo, Greece



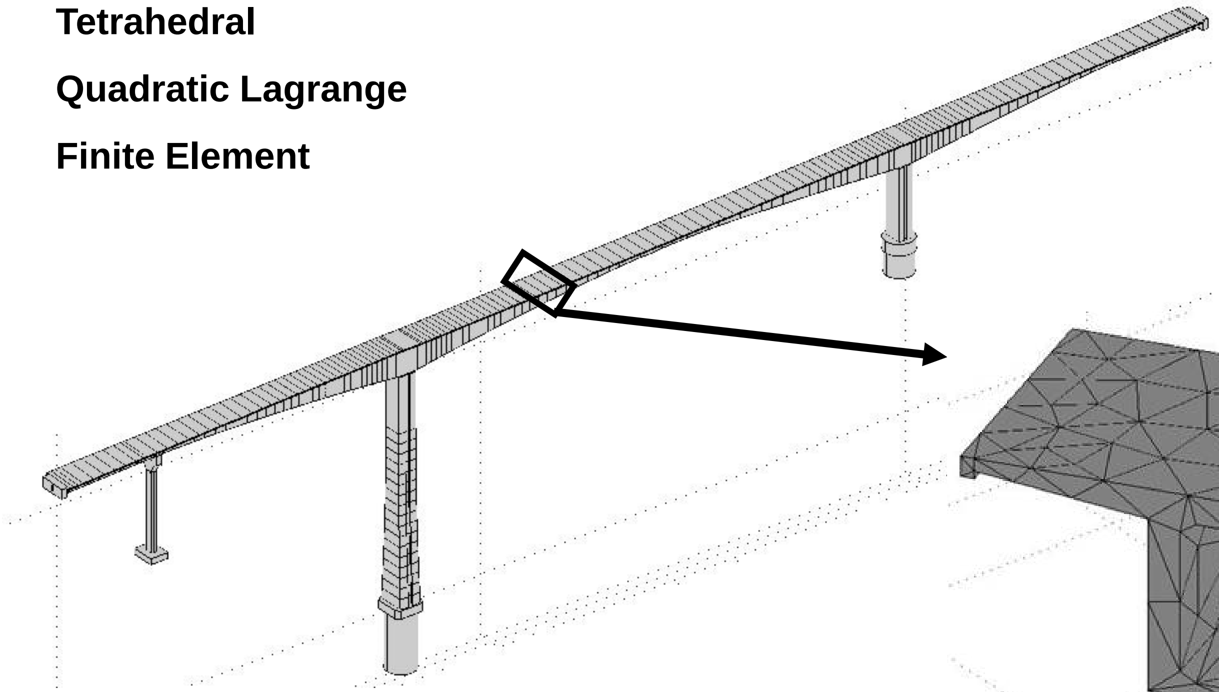
- **Total length: 537m long**
- Deck width: 14m
- M1 height: 45m tall
- **M2 height:110m tall**
- M3 height: 35m tall
- **Central span length: 235m long**

FE Model of Metsovo Bridge

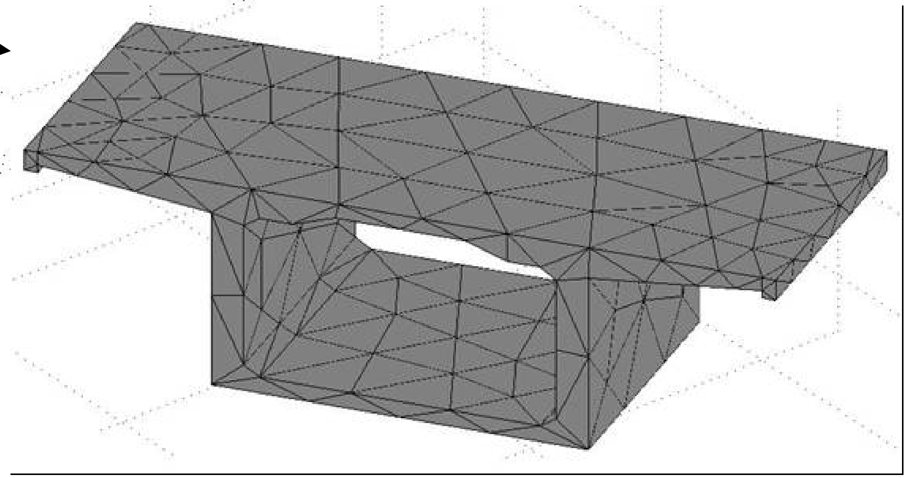
Tetrahedral

Quadratic Lagrange

Finite Element



Extra coarse mesh



DOF: ~ 800,000 - 1,000,000

Largest FE size is limited by

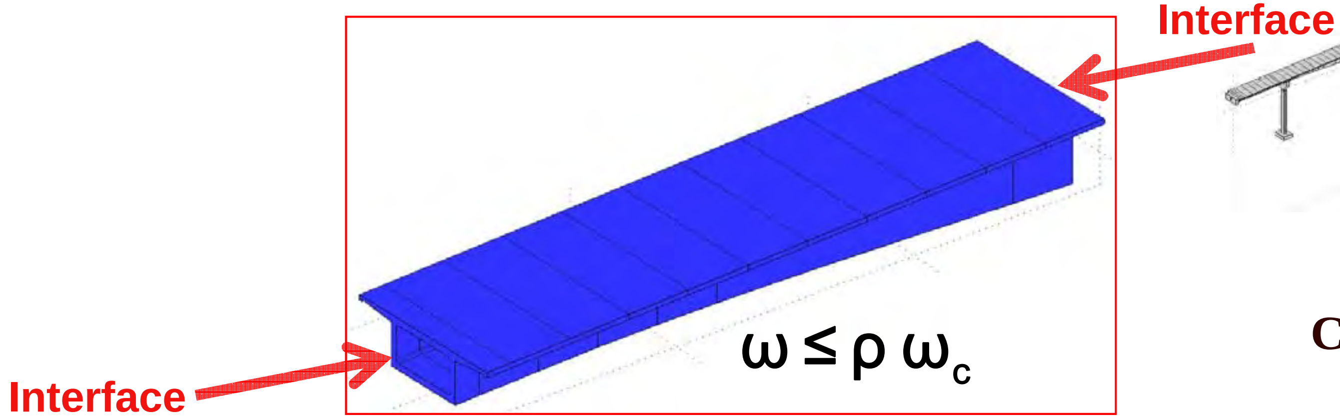
box cross-section thickness.

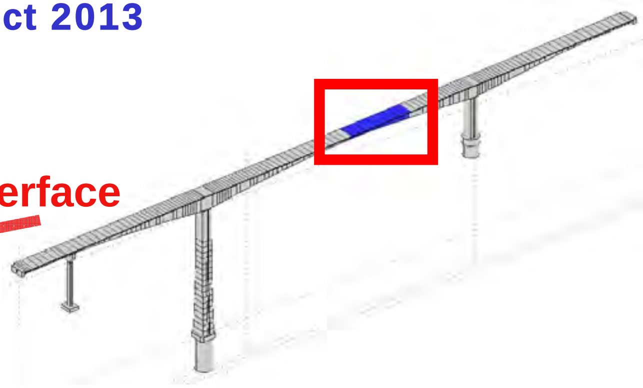
FE size < 0.4m

Length scale (20 modes) > 50m

Component Mode Synthesis (CMS)

Papadimitriou & Papadioti, Comp. & Struct 2013





ω_c

Cut-off Frequency

- **COMPONENT/SUBSTRUCTURE LEVEL**
- **Expand the solution within the component in a reduced base**
  - **Fixed-interface normal modes**

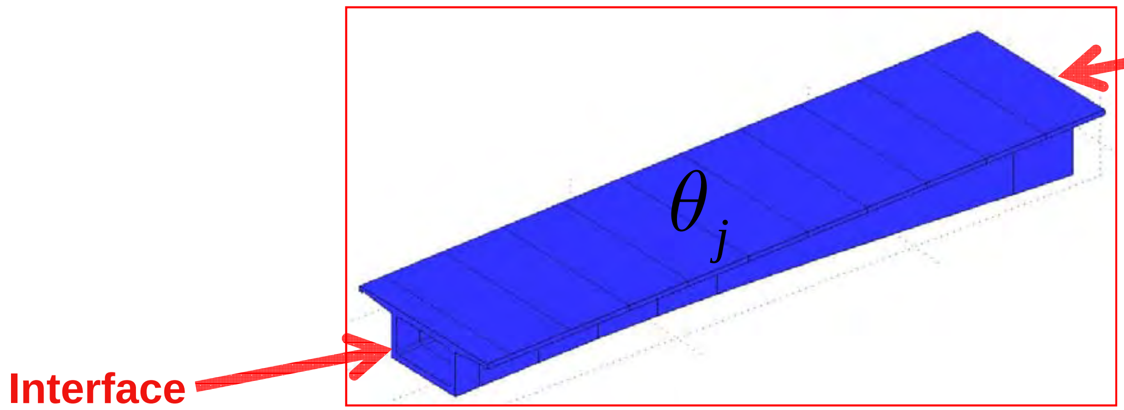
    - Retain a small number of eigenvectors (form the reduced basis) and eigenvalues,

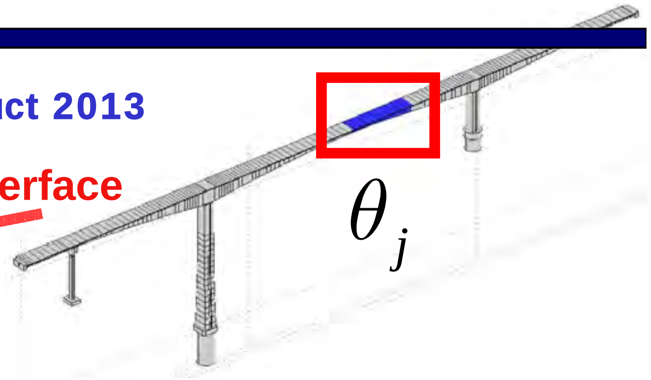
$$\omega \le \rho\, \omega_c, \qquad \rho = 5$$

  - **Interface constrained modes**

Model Updating using CMS

Papadimitriou & Papadioti, Comp. & Struct 2013





- **COMPONENT PARAMETERIZATION**

$$K = \bar{K}\ h(\theta_j) \qquad\qquad M = \bar{M}\ g(\theta_j)$$

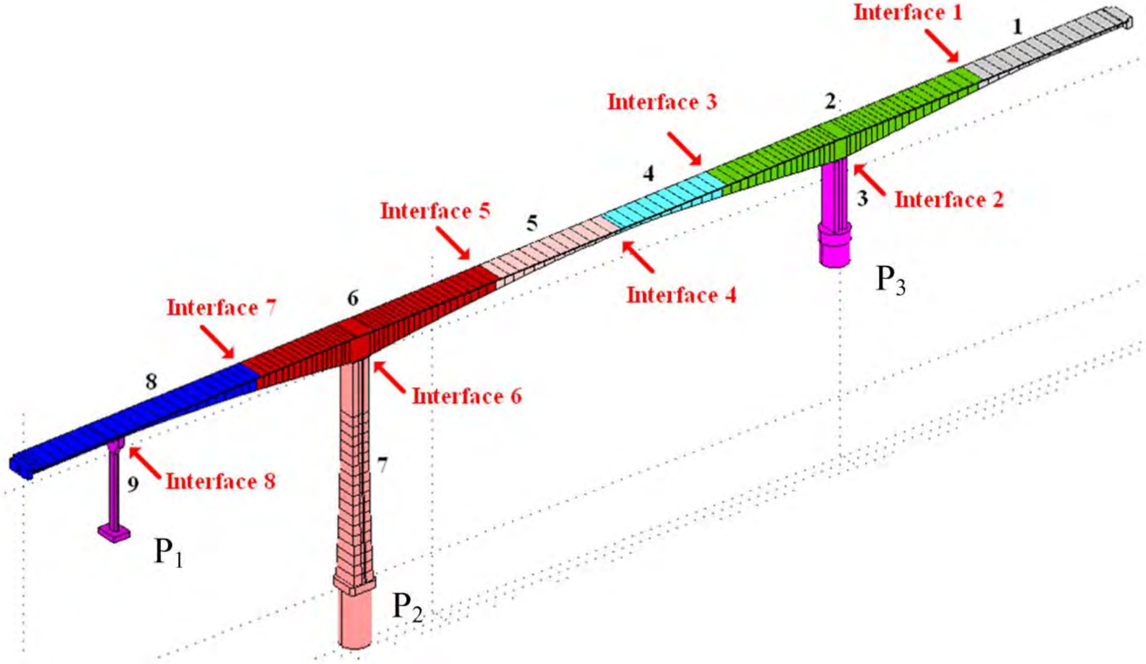
- **Reduced mass and stiffness matrices at system level**

$$K^{CB} = K_0^{CB} + \sum_{i=1}^{N_\theta} K_{1,j}^{CB}\ \frac{h(\theta_j)}{g(\theta_j)} + K_{2,j}^{CB}\ h(\theta_j)$$

$$M^{CB} = M_0^{CB} + \sum_{i=1}^{N_\theta} M_{1,j}^{CB}\ \sqrt{g(\theta_j)} + M_{2,j}^{CB}\ g(\theta_j)$$

Model Updating using CMS

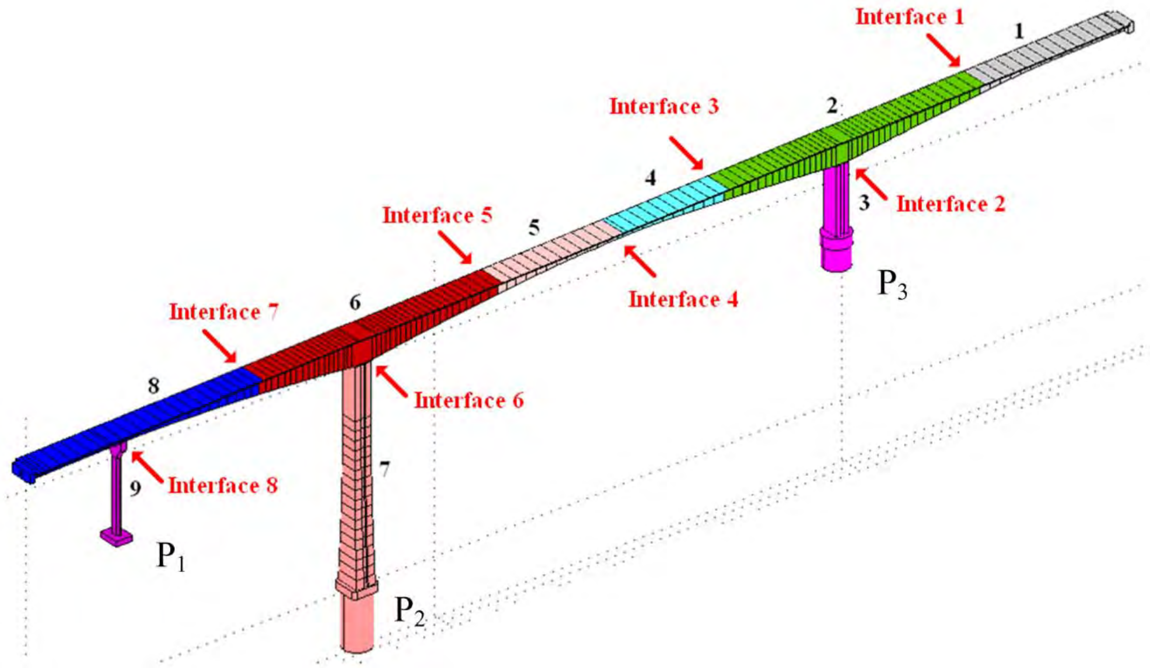
Papadimitriou & Papadioti, Comp. & Struct 2013



- **Reduction of interface DOFs using characteristic interface modes**

  Castanier et al., AIAA 2001

Model Updating using CMS



FE model: ~ 1,000,000 DOF
Reduced model: ~ 500 DOF

Three orders of magnitude reduction of DOF for <0.1% accuracy

20 modes retained in analysis

III.
Application: Metsovo Bridge

1. **Modal Identification**

2. **Finite Element Model Updating**

1. Modal Identification of Metsovo Bridge

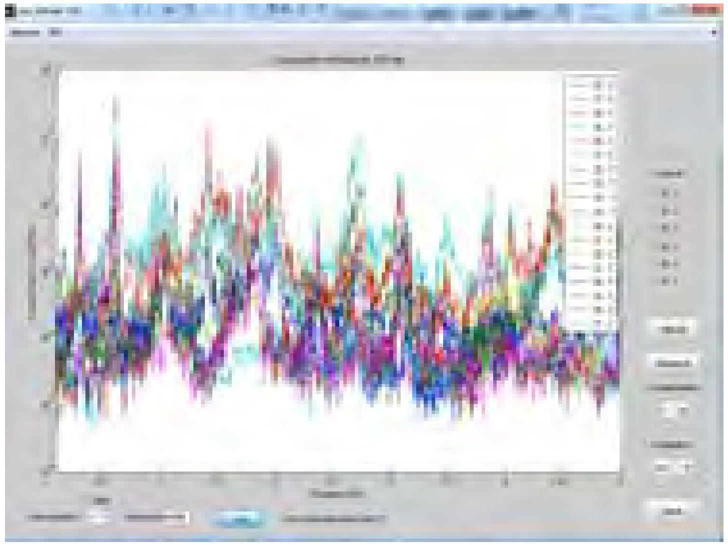
Modal Identification under Ambient Vibrations

Measuring Equipment: 5 triaxial accelerometers
SYSCOM units 3 uniaxial accelerometers (2 horizontal, 1 vertical)

Wireless: GPS for common triggering

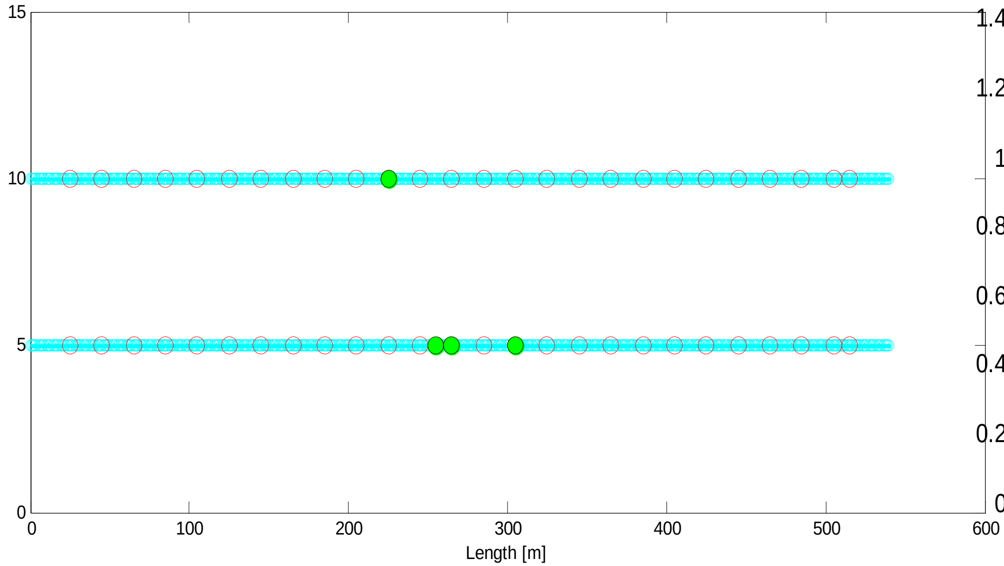
13 different sensor configurations to cover 115 vertical and transverse directions on bridge DECK
3 reference sensors: 2-Vertical, 3-Transverse, 1-Longitudinal

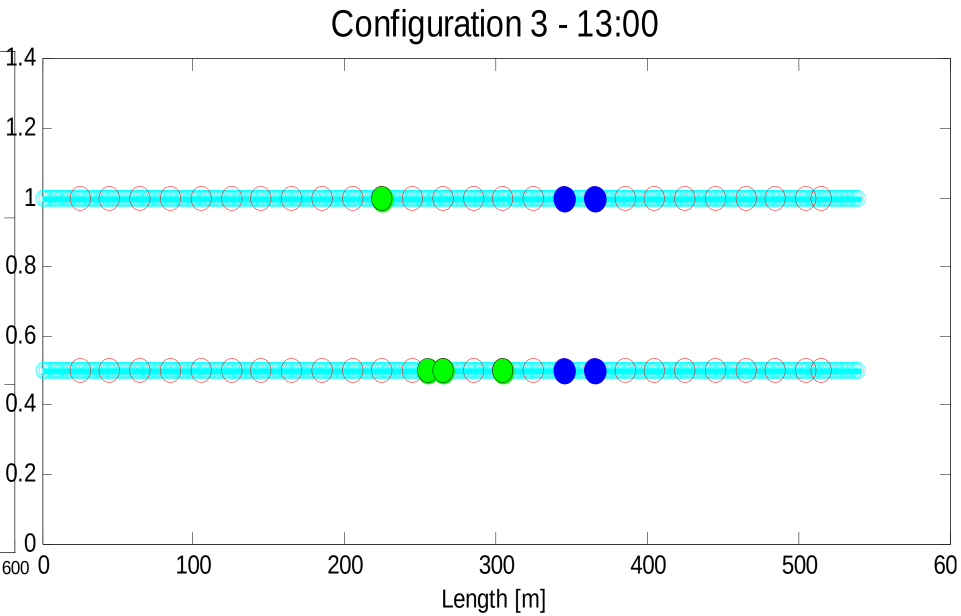
Modal Identification Toolbox: MI-Tool
University of Thessaly



Multiple Sensor Configurations

•Acceleration time histories were collected from 5 tri-axial and 3 uni-axial sensors under normal operating conditions of the bridge.
•Excitation was traffic ranging from light vehicles to heavy trucks, and environmental excitation such as ground micro-tremor and wind loading.
•13 sensor configurations were used to cover the entire length in steps of 20m
•Data acquisition time was 20min, and 10min break to move the sensors.
•GPS time synchronization was used to start recording simultaneously in all sensors.

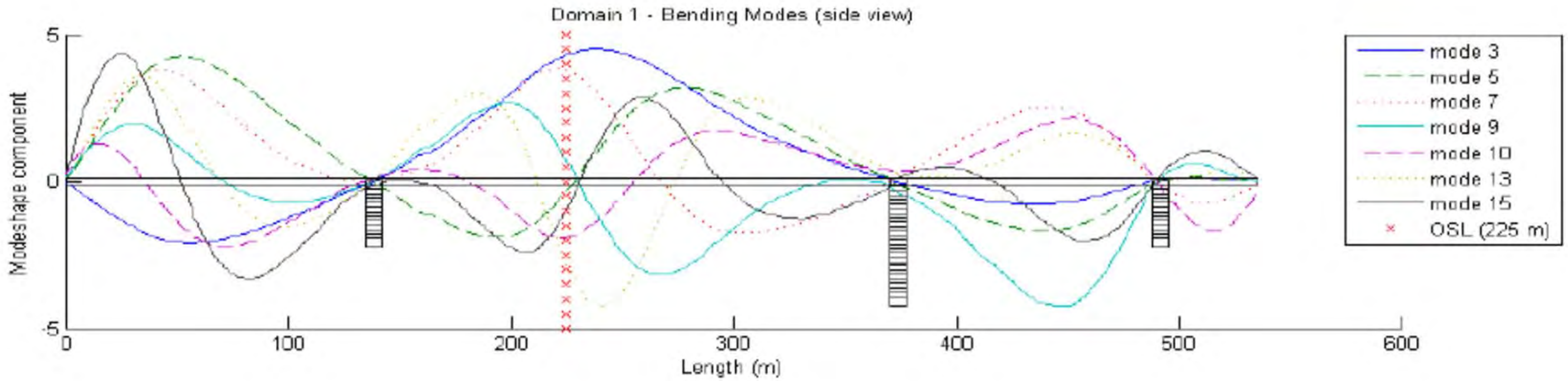




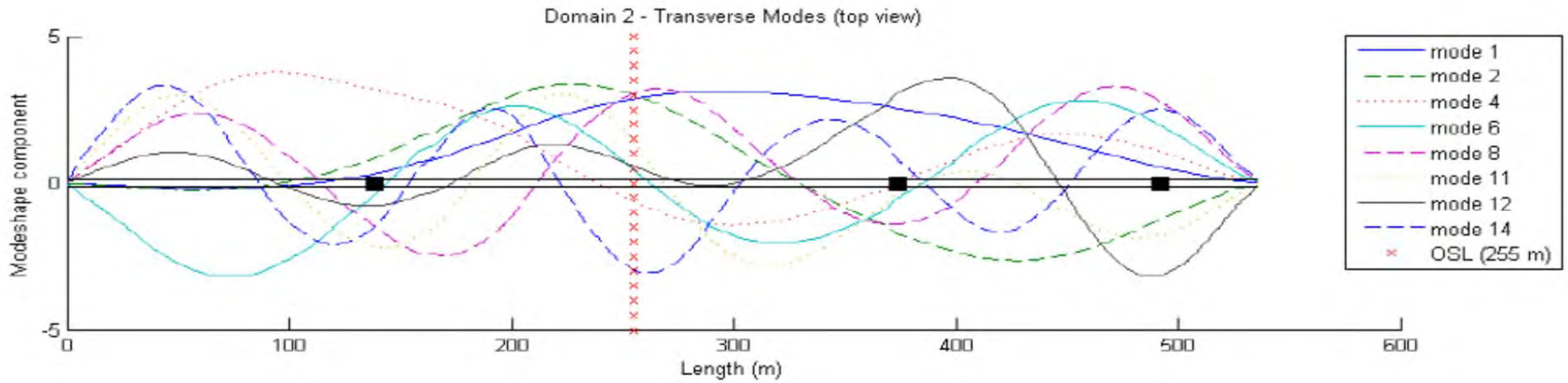
Optimal Sensor Placement of reference sensors

The locations of the reference sensors were optimized in order to avoid nodes of mode shapes and maximize useful information gain for mode shape identification. The Optimal Sensor Location theory used is based on the principle of maximum entropy. **[Papadimitriou, Beck and Au, J. Vibration & Control 2000]**

Vertical Reference Sensor

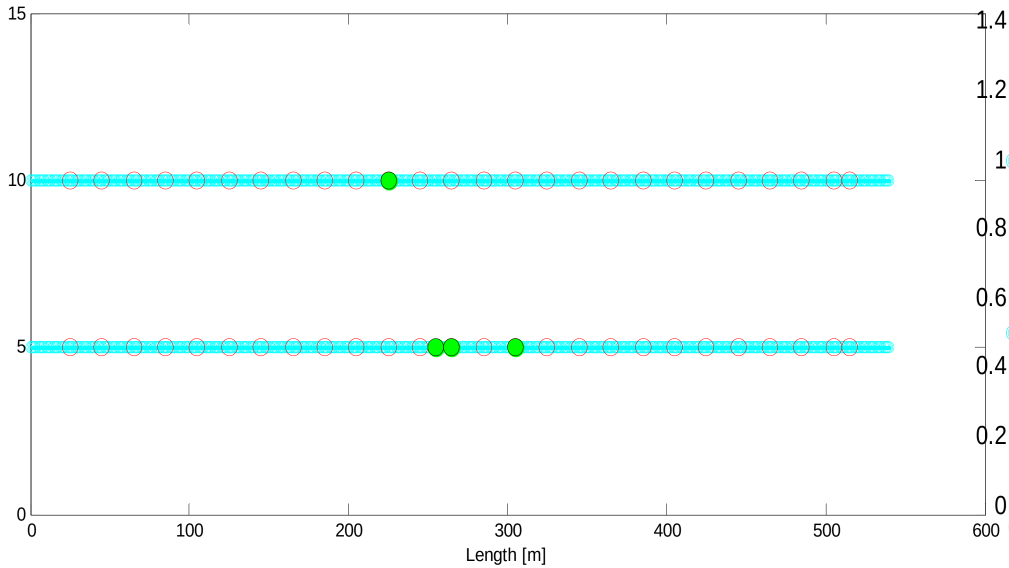


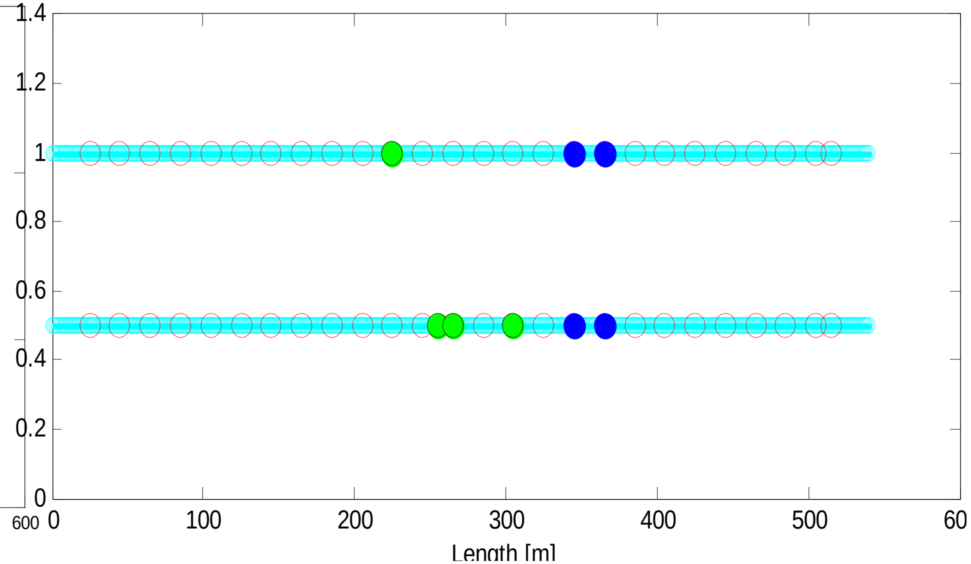
Transverse Reference Sensor



Multiple Sensor Configurations

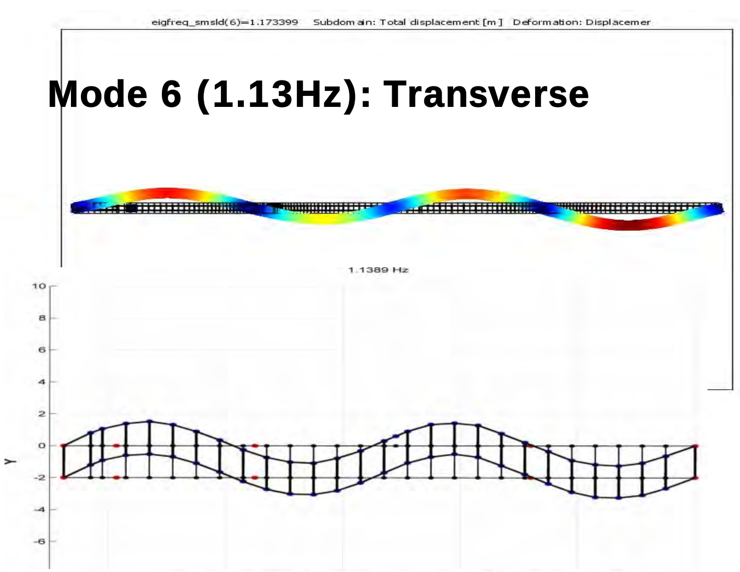
•The Optimal Sensor location theory for the reference sensors is based on the F.E. model. In order to be safe, one more reference sensor was used.
•The moving sensors covered the entire length in steps of 20m.
•The identification of modal properties from ambient acceleration data is done based on fitting the Power Spectra of the time histories.
•The measurements from all 13 sensor configurations were imported simultaneously into the software to yield the modal properties.
•The mode shape assembly methodology used by the software to assemble the global mode shape from the local ones is based on a least-squares approach. **S. K. Au. Assembling mode shapes by least squares. *Mechanical Systems and Signal Processing*, 25, 163-179, 2010.**



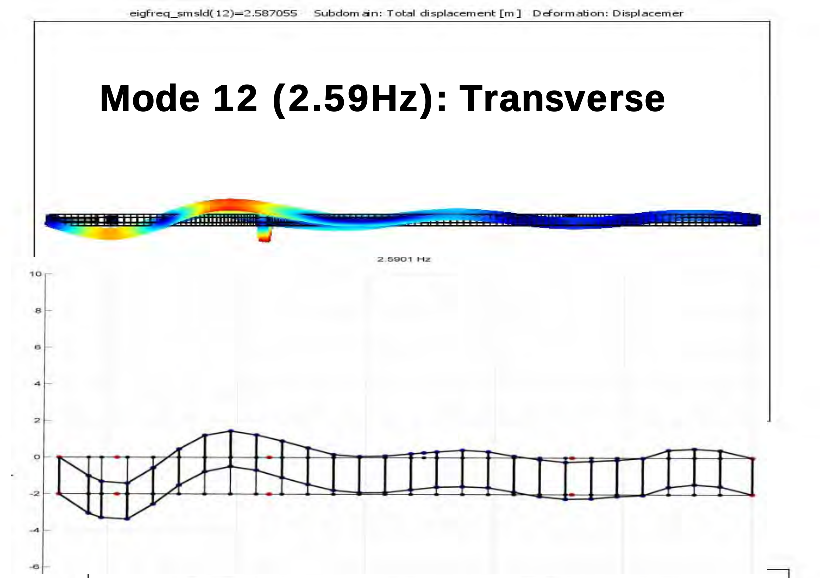


Model & Experimentally Identified Modeshapes

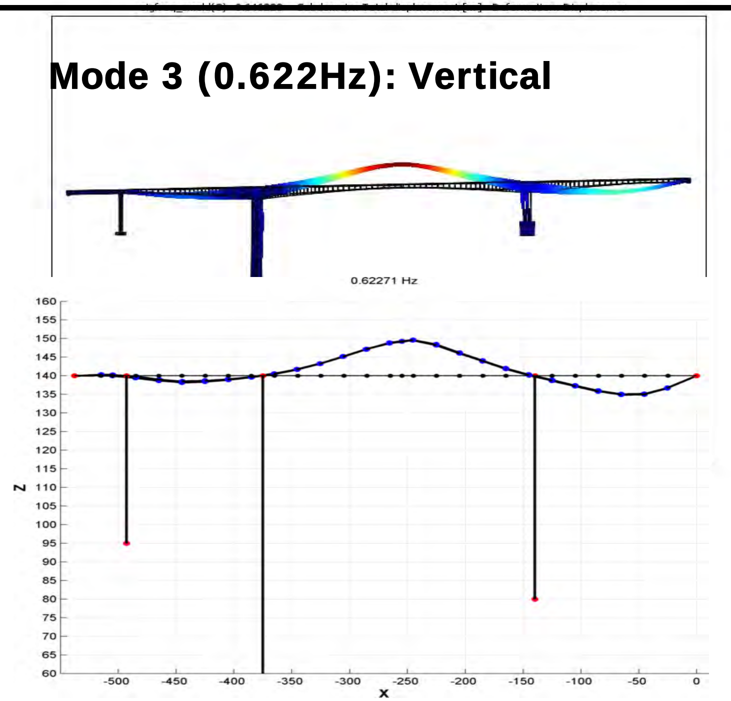
Mode 6 (1.13Hz): Transverse



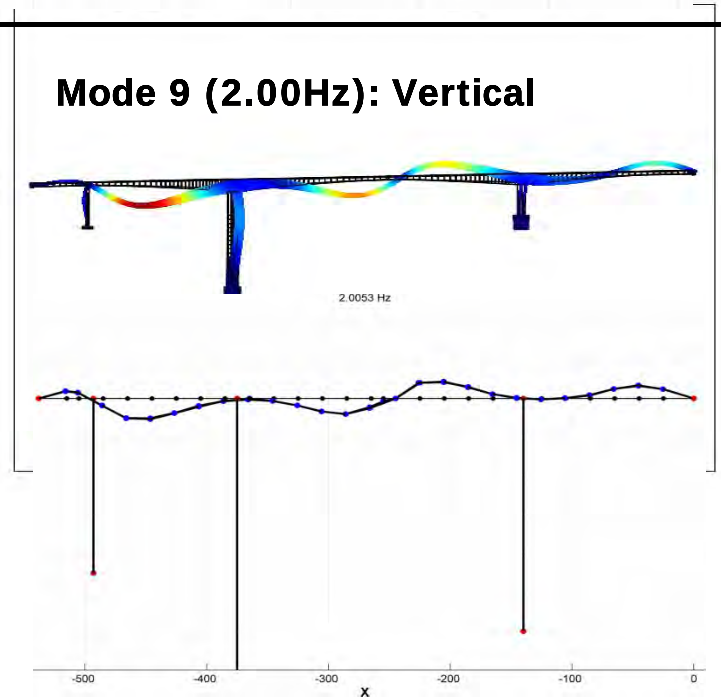
Mode 12 (2.59Hz): Transverse



Mode 3 (0.622Hz): Vertical



Mode 9 (2.00Hz): Vertical



2. Finite Element Model Updating of Metsovo Bridge

- **Data:** Modal frequencies and mode shapes
- **Analysis:** Parameter estimation
- **Model:** Linear with very large number of DOFs
- **Technique:** Model reduction (CMS)
- **Bayesian Tool:** X-TMCMC

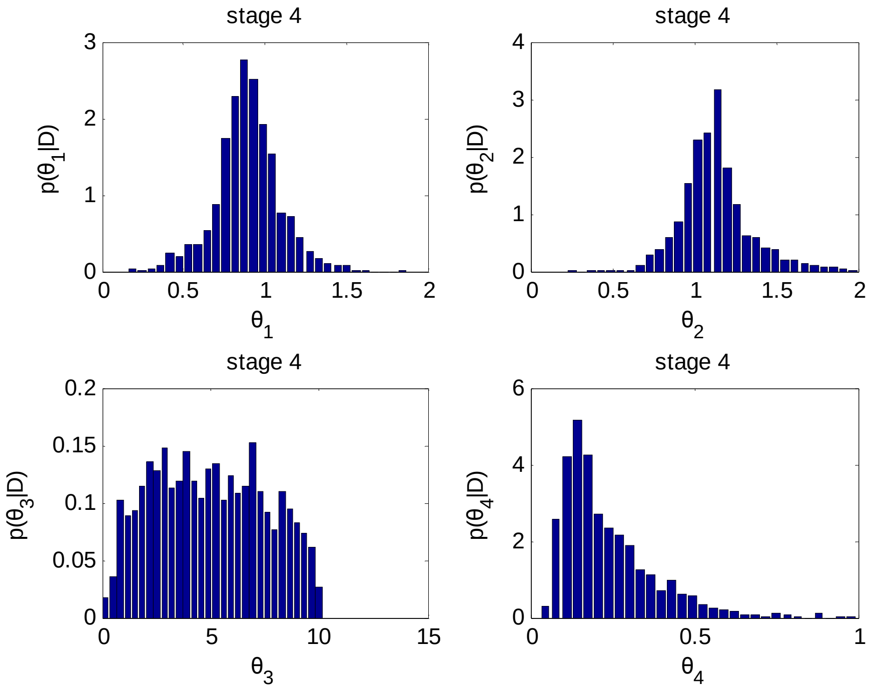
Model Updating

Comparison of Modal Frequencies

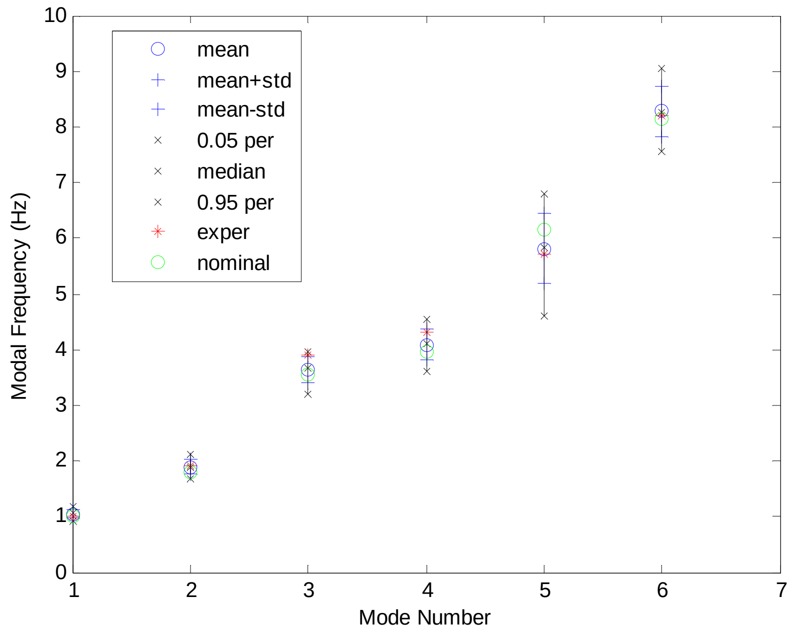
Mode	Type	Exp. Mean	Exp. STD	Nominal	Model Mean	Model STD
1	Transverse	0,3063	0,0007	0,3180	0,3076	0,0049
2	Transverse	0,6034	0,0014	0,6220	0,5960	0,0071
3	Bending	0,6227	0,0008	0,6460	0,6236	0,0081
4	Transverse	0,9646	0,0084	0,9890	0,9227	0,0276
5	Bending	1,0468	0,0066	1,1120	1,1095	0,0107
6	Transverse	1,1389	0,0049	1,1730	1,1381	0,0117
7	Bending	1,4280	0,0042	1,5160	1,4664	0,0167
8	Transverse	1,6967	0,0112	1,7110	1,6444	0,0172
9	Bending	2,0053	0,0054	1,9340	1,8322	0,0203
10	Transverse	2,3666	0,0025	2,3520	2,2192	0,0252

Model Updating

Parameter Estimation

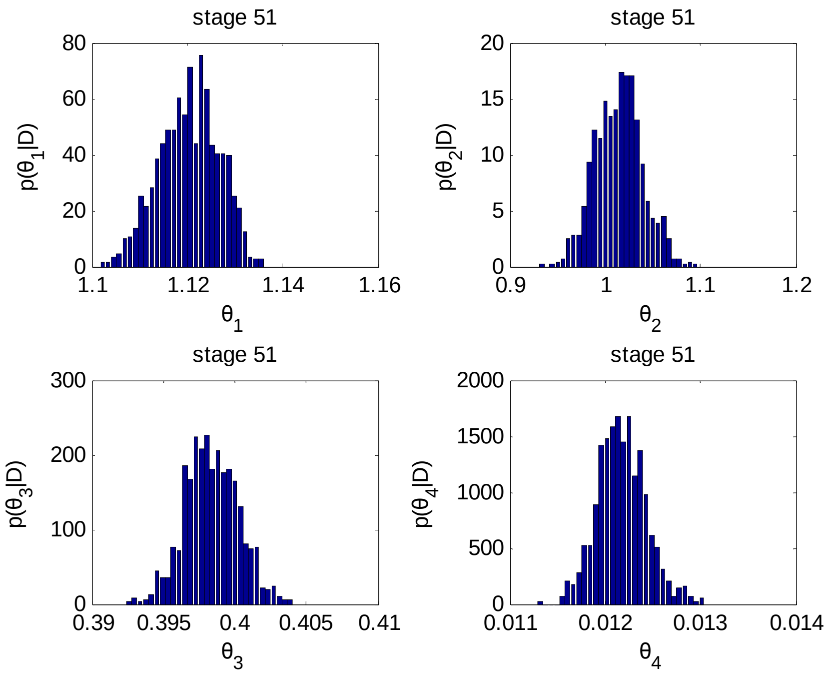


Modal Frequency Fit



Model Updating

Parameter Estimation



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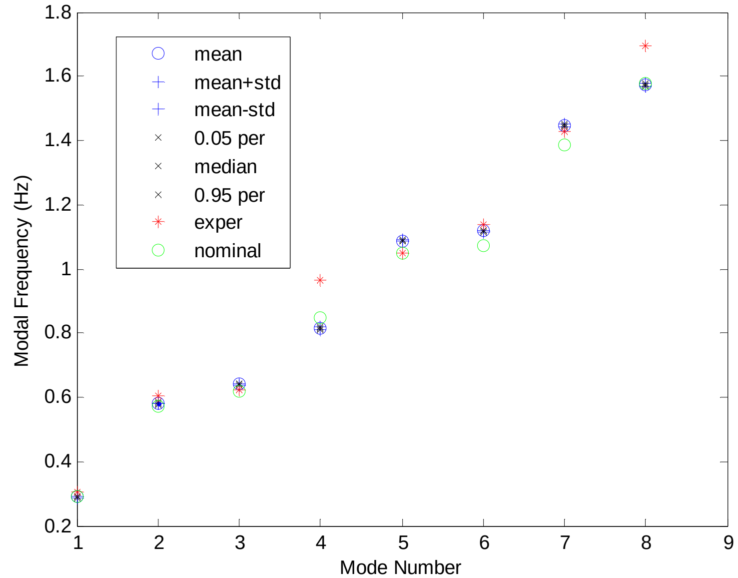
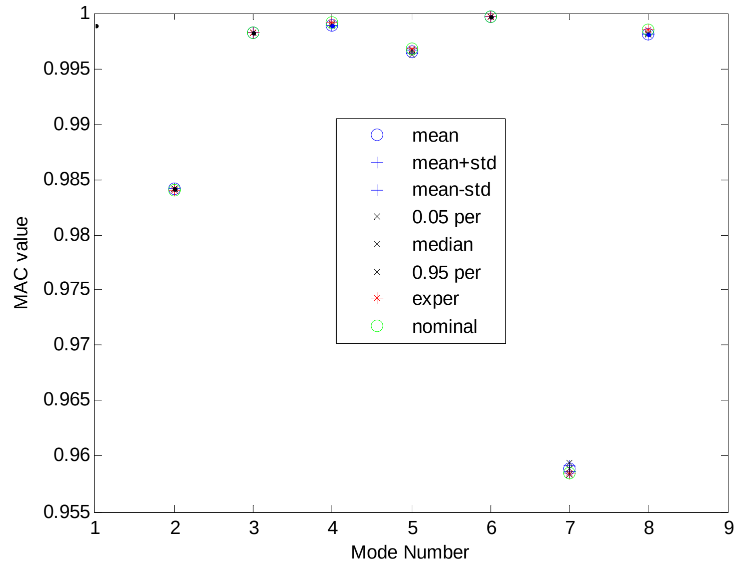
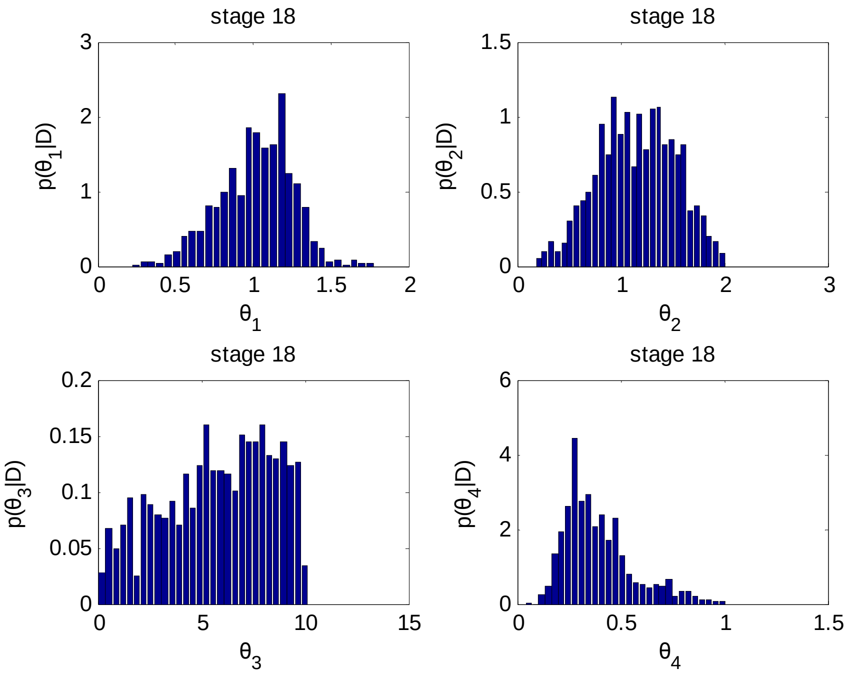


MAC

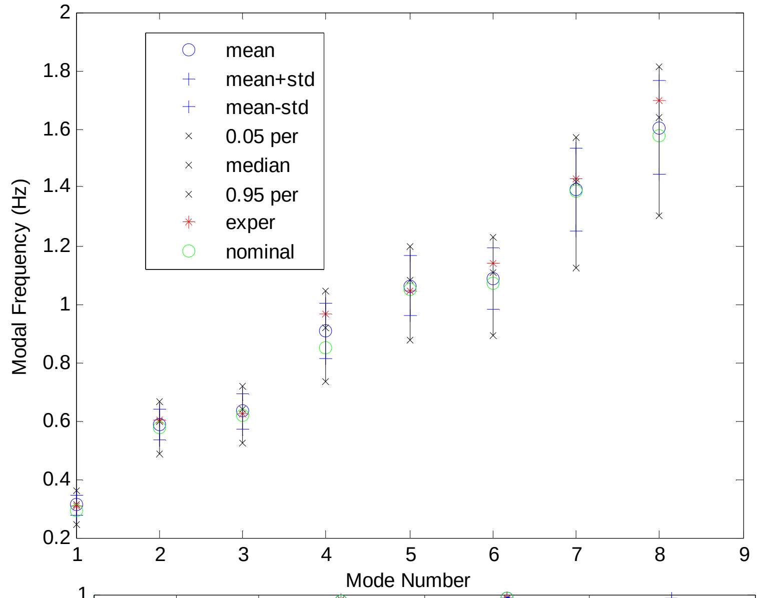


Model Updating

Parameter Estimation



Modal Frequency Fit



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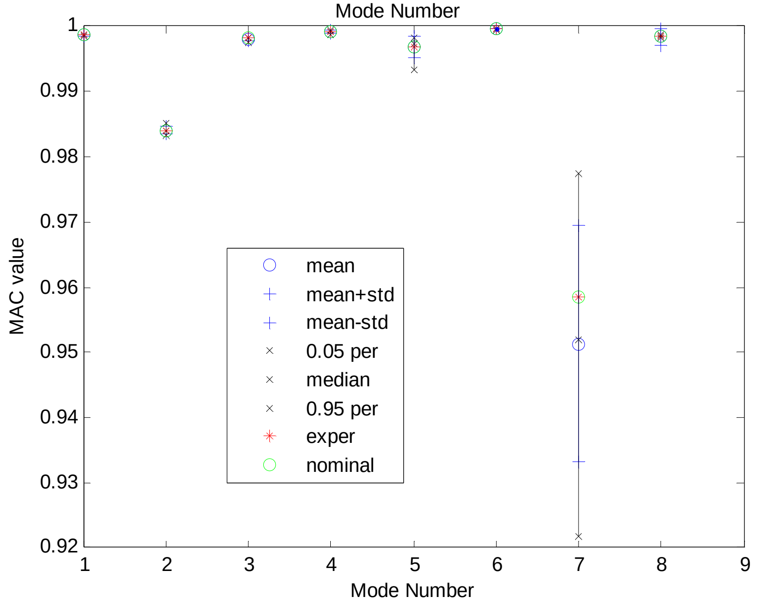
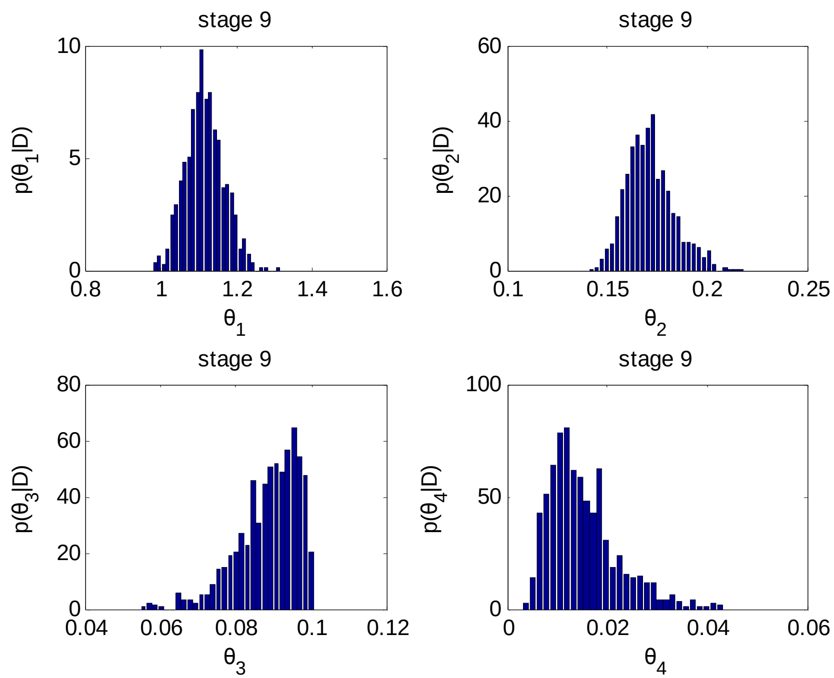
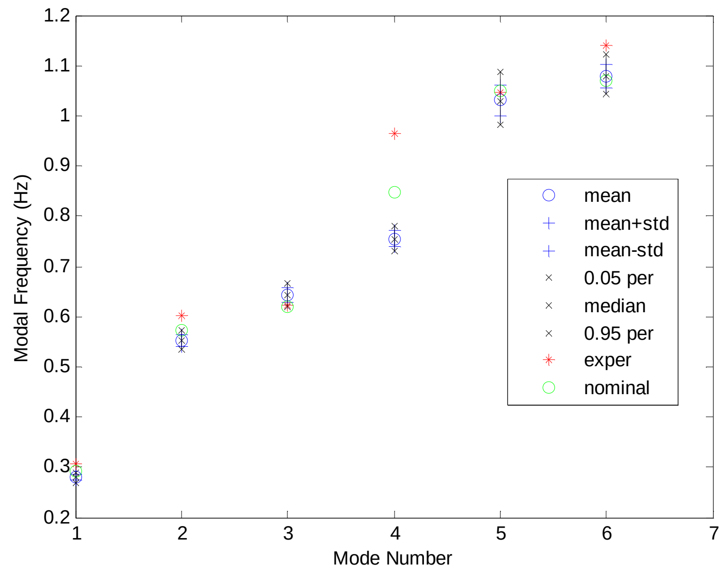


Model Updating

Parameter Estimation



Modal Frequency Fit



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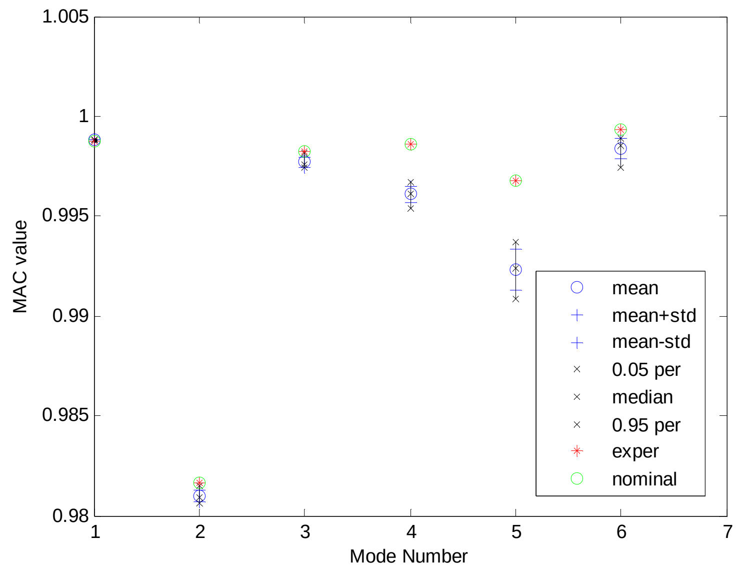

Conclusions

- In HPC environments, **stochastic optimization algorithms (e.g. CMA-ES) and multi-chain MCMC (e.g. Transitional MCMC)** have certain advantages (convenience, accuracy and computational efficiency) for Bayesian FE model updating

- **Π4U** general purpose software for Bayesian UQ+P using non-intrusive models [http://www.cse-lab.ethz.ch/software/Pi4U]
  - Π=Παραλληλισμός (Parallelism)
  - Π=Πιθανότητα (Probability)
  - Π=Ποσοτικοποίηση (Quantification)
  - Π=Πρόβλεψη (Prediction)
  - U=Uncertainty

  [Hadjidoukas, Angelikopoulos, Papadimitriou, Koumoutsakos, Journal of Computational Physics, 2015]